HEAT TRANSFER BETWEEN AN EDDYING FLOW AND A THREE-DIMENSIONAL HEAT SOURCE

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The heat transfer of eddying flows is interesting in connection with the development and operation of various types of energy devices which employ local swirling of a flow. Many of the features of such flows were examined in [1-3]. The studies [4-6] were devoted to the numerical modeling of the hydrodynamics of internal eddying flows.

Here, we conduct a numerical study of the effect of swirling on the heat transfer of an eddying laminar flow with a heat source of constant intensity. Flows with such heat sources are used in a number of engineering devices [7]. Moreover, the use of a heat source of constant intensity makes it possible to model the heat transfer of a chemically reactive flow with a low thermal efficiency [8].

With the use of the stream function and vorticity as variables, we can write the system of equations which describes the incompressible flow and heat transfer of such a flow in dimensionless form

$$\frac{\mathrm{Re}}{4}\xi^2\left\{\frac{\partial}{\partial x}\left[\frac{\omega}{\xi}\frac{\partial\psi}{\partial\xi}\right]-\frac{\partial}{\partial\xi}\left[\frac{\omega}{\xi}\frac{\partial\psi}{\partial x}\right]\right\}-\left\{\frac{\partial}{\partial x}\left[\xi^3\frac{\partial(\omega/\xi)}{\partial x}\right]+\right.$$
$$\left.+\frac{\partial}{\partial\xi}\left[\xi^3\frac{\partial(\omega/\xi)}{\partial\xi}\right]\right\}-\mathrm{Re}\,\xi\frac{\partial v_\varphi^2}{\partial x}=0, \tag{1}$$
$$\frac{\partial}{\partial x}\left[\frac{1}{\xi}\frac{\partial\psi}{\partial x}\right]+\frac{\partial}{\partial\xi}\left[\frac{1}{\xi}\frac{\partial\psi}{\partial\xi}\right]+\omega=0,$$
$$\frac{\mathrm{Re}}{4}\left\{\frac{\partial}{\partial x}\left[v_\varphi\xi\frac{\partial\psi}{\partial\xi}\right]-\frac{\partial}{\partial\xi}\left[v_\varphi\xi\frac{\partial\psi}{\partial x}\right]\right\}-\left\{\frac{\partial}{\partial x}\left[\xi\frac{\partial(\xi v_\varphi)}{\partial x}\right]+\right.$$
$$\left.+\frac{\partial}{\partial\xi}\left[\xi\frac{\partial(\xi v_\varphi)}{\partial\xi}\right]\right\}+2\frac{\partial(v_\varphi\xi)}{\partial\xi}=0,$$
$$\frac{\mathrm{Pe}}{4}\left\{\frac{\partial}{\partial x}\left[\theta\frac{\partial\psi}{\partial\xi}\right]-\frac{\partial}{\partial\xi}\left[\theta\frac{\partial\psi}{\partial x}\right]\right\}-\left\{\frac{\partial}{\partial x}\left[\xi\frac{\partial\theta}{\partial x}\right]+\frac{\partial}{\partial\xi}\left[\xi\frac{\partial\theta}{\partial\xi}\right]\right\}-S_T\xi=0.$$

In assigning the boundary conditions, we assumed that the temperature of the flow in the transverse direction was constant at the inlet of the tube and that the velocity distribution conformed to the law for a solid:

$$\psi=\xi^2,\ \omega=0,\ v_\varphi=\sigma\xi,\ \theta=0 \qquad \text{at } x=0. \tag{2}$$

At the outlet, we imposed mild conditions modeling the free discharge of fluid:

$$\partial^2\psi/\partial x^2=0,\ \partial^2\omega/\partial x^2=0,\ \partial^2 v_\varphi/\partial x^2=0 \qquad \text{at } x=L. \tag{3}$$

On the axis of the tube, the following symmetry conditions were valid:

$$\psi=0,\ v_\varphi=0,\ \partial\theta/\partial\xi=0 \qquad \text{at } \xi=0. \tag{4}$$

On the wall of the tube, we monitored the constancy of the flow rate. Here, we simulated "adhesion" conditions and assumed that the heat-conducting properties of the wall were ideal:

$$\psi=1,\ v_\varphi=0,\ \theta=0 \qquad \text{at } \xi=1. \tag{5}$$

Here, $\omega = 2(\partial v_\xi/\partial x - \partial v_x/\partial\xi)$ is the dimensionless vorticity; ψ is the stream function; θ is dimensionless temperature; $\sigma = \Omega R/U$ is a parameter characterizing the intensity of swirling at the inlet of the tube; Ω is the angular velocity at x = 0; $U = 2R^{-2}\int_0^R v_x r\,dr$ is the mean flow velocity; R is the radius of the tube; $\mathrm{Re} = 2UR/\nu$, $\mathrm{Pe} = 2UR/\kappa$ are the Reynolds and Peclet numbers; S_T = const is the heat source.

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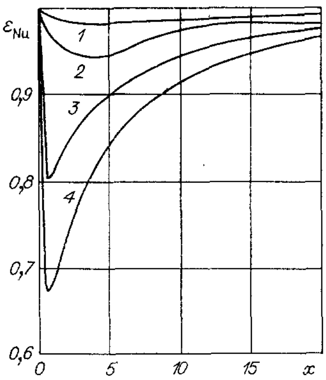

Fig. 1

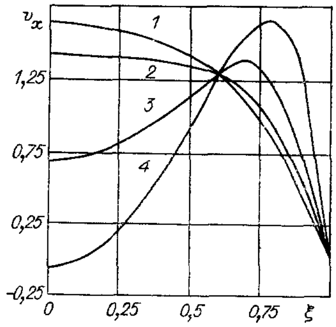

Fig. 2

We used the finite-difference method in [9] to solve system (1) with boundary conditions (2). This method is sufficiently simple and universal. The problem was solved on a 31 × 15 coordinate grid that was nonuniform with respect to both coordinates. The points of the grid were crowded at the inlet and near the wall of the tube. The finite-difference analog of the system of differential equations is a system of nonlinear algebraic equations that was solved by the Gauss–Seidel method. Convergence of the iterations at $\sigma \geq 3$ was assured by using lower relaxation for the vorticity and circulation $v_\varphi \xi$. The criterion of convergence was satisfaction of the inequality $|1 - \varphi^{(N-1)}/\varphi^{(N)}| < 10^{-3}$ $(\varphi = (\psi, \omega/\xi, \xi v_\varphi, \theta)$, N is the number of the iteration).

The vorticity on the axis and wall of the tube were determined by the method in [9]. The axial and radial components of velocity were found by numerical differentiation of the stream function (in a three-point scheme):

$$v_x = \frac{1}{2\xi}\frac{\partial\psi}{\partial\xi}, \quad v_\xi = -\frac{1}{2\xi}\frac{\partial\psi}{\partial x}.$$

Calculations performed for $\sigma < 6$ in the range $10^2 \leq \mathrm{Re} \leq 10^3$ show that the change in the fields of the axial and tangential components of velocity and temperature with variation of Re reduces to a coordinate transformation which increases the longitudinal coordinate x by the factor Re. Thus, the parametric relations $v_x = v_x(x, \xi, \mathrm{Re}, \sigma)$ $v_\varphi = v_\varphi(x, \xi, \mathrm{Re}, \sigma)$, $\theta = \theta(x, \xi, \mathrm{Re}, \sigma)$ can be reduced to relations of the form $v_x = v_x(z, \xi, \sigma)$, $\theta = \theta(z, \xi, \sigma)$, where $z = x\mathrm{Re}^{-1}$. For the radial component of velocity, the transformation $v_\xi = \mathrm{Re} f(z, \xi, \sigma)$.

Figure 1 shows the downstream change in the relative heat-transfer coefficient $\varepsilon = \mathrm{Nu}/\mathrm{Nu}_0$ for Re = 160 and different values of swirling intensity σ ($\mathrm{Nu} = \langle\theta\rangle^{-1}\partial\theta/\partial\xi|_{\xi=1}$ corresponds to eddying flow, Nu_0 corresponds to forward flow $\sigma = 0$, and $\langle\theta\rangle = 2\int_0^1 v_x\theta\xi d\xi$ corresponds to the mean-flow-rate temperature). Curves 1-4 correspond to σ = 2, 3, 4, and 6, respectively. It is evident that swirling leads to a deterioration in heat transfer. Here, an increase in σ is accompanied by a decrease in ε over the entire flow region. Downstream, as the swirling decays, the value of Nu obtained for eddying flows approaches the values of Nu for forward flows.

Let us examine the reasons for such a heat-transfer regime. There are several distinctive features to the spontaneous heating of the flow with an internal three-dimensional source of heat in the initial flow section. One of these features is the dependence of the rate of heating on the velocity distribution in the flow. The faster-moving layers of fluid are carried downstream without being significantly heated. Conversely, the slower layers travel less distance before undergoing heating. Thus, in the case of an adiabatic wall $(\partial\theta/\partial\xi|_{\xi=1} = 0)$, the temperature reaches the maximum at $\xi = 1$, where $v_x = 0$. Heat transfer lowers the temperature of the wall region, so that the coordinate of the point with the maximum value of temperature in the section will be found at $\xi_* < 1$.

The velocity distribution in the flow acquires a more complicated form with the occurrence of swirling. The latter results in unusual temperature profiles. Figure 2 shows the distribution of axial velocity in the section x = 3, while Fig. 3 shows the temperature distribution in this section for Re = 160, S_T = 10. Curve 1 is for forward flow with σ = 0, while curves 2-4 are for swirled flow with σ = 2, 4, 6.

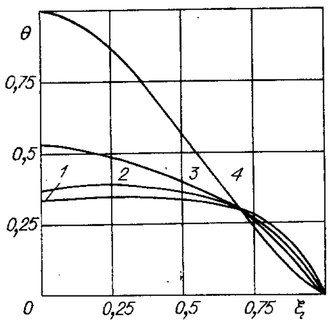

Fig. 3

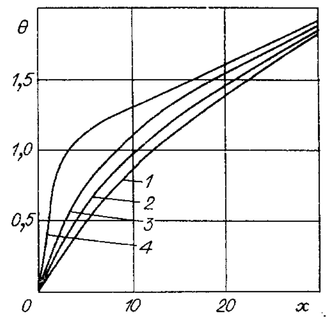

Fig. 4

At $\sigma \leq 1$, the effect of centrifugal forces on the motion and heat transfer of the flow is negligible: the velocity profiles and temperature distributions with $\sigma = 0$ and 1 are similar. With an increase in swirling ($\sigma \geq 2$), centrifugal forces result in the formation of a low-pressure region in the axial zone. This in turn results in the formation of a "trough" of v_x in the core of the flow (up until the appearance of recirculation flows at $\sigma \geq 6$) and the appearance of a maximum of axial velocity at a certain $\xi = \xi_m \neq 0$ (Fig. 2). An increase in swirling is accompanied by an increase in the maximum value of v_x and shifting of ξ_m toward the wall. Thus, along with the usual factor which impedes heating of the wall region (heat removal), there is another factor in effect – an increase in flow rate near the wall. This leads to a situation whereby an increase in swirling is accompanied by a decrease in temperature in the wall region. The temperature profile becomes shallower near the wall (Fig. 3), while the temperature gradient $\partial\theta/\partial\xi|_{\xi=1}$ decreases in absolute value.

The slowing of the flow in the axial region in the case of swirling leads to more intensive heating of this region. Thus, the temperature on the tube axis increases with swirling. Figure 4 shows the change in $\theta(x, 0)$ downstream for $\sigma = 0, 3, 4$, and 6 (lines 1-4) with Re = 160, $S_T = 10$. Downstream, where the effect of centrifugal forces becomes weaker, the temperature curves approach one another. The increase in temperature in the axial zone is almost completely offset by the reduction in temperature in the wall region. Thus, we see only a slight increase in $\langle\theta\rangle$ (on the order of 3-4%).

Consequently, the effect of swirling on the gradient $\partial\theta/\partial\xi|_{\xi=1}$ and the mean-flow-rate temperature $\langle\theta\rangle$ leads, with a fixed flow rate, to a decrease in Nu as swirling increases. This result, somewhat surprising at first glance, must be considered in the use of heat exchangers which employ eddying flows with an internal heat source.

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